GOAL

Identify and use translations in the plane.

VOCABULARY

A translation is a transformation that maps every two points $P$ and $Q$ in the plane to points $P'$ and $Q'$, so that the following properties are true:
1) $PP' = QQ'$ and 2) $P'P || Q'Q$, or $PP'$ and $QQ'$ are collinear.

A vector is a quantity that has both direction and magnitude, or size.

When a vector is drawn as ray $\overrightarrow{PQ}$, the initial point, or starting point, of the vector is point $P$ and the terminal point, or ending point, of the vector is point $Q$.

The component form of a vector combines the horizontal and vertical components.

EXAMPLE 1

Using Theorem 7.5

In the diagram, a reflection in line $k$ maps $\overline{AB}$ to $\overline{A'B'}$, a reflection in line $m$ maps $\overline{A'B'}$ to $\overline{A''B''}$, $k || m$, $AW = 7$, and $ZA'' = 3$.

a. Name some congruent segments.
c. What is the length of $\overline{AA''}$?

SOLUTION

a. Here are some sets of congruent segments: $\overline{AB}$, $\overline{A'B'}$, and $\overline{A''B''}$; $\overline{BX}$ and $\overline{X'B'}$, $\overline{BY}$ and $\overline{Y'B''}$.

b. Yes, $WZ = XY$ because $WZ$ and $XY$ are opposite sides of a rectangle.

c. Because $AA'' = BB''$, the length of $\overline{AA''}$ is $7 + 7 + 3 + 3$, or 20 units.
Practice with Examples

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**Exercise for Example 1**

In the diagram $k \parallel m$, $\triangle XYZ$ is reflected in line $k$, and $\triangle X'Y'Z'$ is reflected in line $m$.

1. Name two segments parallel to $YY''$.

2. If the length of $ZZ''$ is 6 cm, what is the distance between $k$ and $m$?

3. A translation maps $\triangle XYZ$ onto which triangle?

4. Which lines are perpendicular to $XX''$?

**Example 2**

**Translations in a Coordinate Plane**

Sketch a quadrilateral with vertices $A(0, 4), B(-2, 1), C(0, -3),$ and $D(3, 4)$. Then sketch the image of the quadrilateral after the translation $(x, y) \rightarrow (x + 2, y - 1)$.

**Solution**

Plot the points as shown. Shift each point 2 units to the right and 1 unit down to find the translated vertices.

**Exercises for Example 2**

In Exercises 5–8, copy figure $PQRS$ and draw its image after the translation.

5. $(x, y) \rightarrow (x - 4, y + 1)$
6. $(x, y) \rightarrow (x, y - 5)$
Finding Vectors

In the diagram, \( \triangle ABC \) maps onto \( \triangle A'B'C' \) by a translation. Write the component form of the vector that can be used to describe the translation.

**Solution**

Choose any vertex and its image, say \( A \) and \( A' \). To move from \( A \) to \( A' \), you move 3 units to the right and 5 units down. The component form of the vector is \( (3, -5) \).

**Exercises for Example 3**

In Exercises 9 and 10, write the component form of the vector that describes the translation which maps \( \triangle ABC \) onto \( \triangle A'B'C' \).

9. \( A(3, 6), B(1, 0), C(4, 8); A'(1, 2), B'(-1, -4), C'(2, 4) \)

10. \( A(-6, -2), B(-5, 3), C(1, -1); A'(-3, -5), B'(-2, 0), C'(4, -4) \)